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To cite this article: Aysun Yurttas Gunes , Musa Demirci , Mert Sinan Oz & Ismail Naci Cangul (2020) Matching number and characteristic polynomial of a graph, Journal of Taibah University for Science, 14:1, 993-999, DOI: [10.1080/16583655.2020.1796275](https://doi.org/10.1080/16583655.2020.1796275)

To link to this article: <https://doi.org/10.1080/16583655.2020.1796275>



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Published online: 25 Jul 2020.



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Matching number and characteristic polynomial of a graph

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ABSTRACT

Matching number and the spectral properties depending on the characteristic polynomial of a graph obtained by means of the adjacency polynomial has many interesting applications in different areas of science. There are some work giving the relation of these two areas. Here the relations between these two notions are considered and several general results giving this relations are obtained. A result given for only unicyclic graphs is generalized. There are some methods for determining the matching number of a graph in literature. Usually nullity, spanning trees and several graph parts are used to do this. Here, as a new method, the conditions for calculating the matching number of a graph by means of the coefficients of the characteristic polynomial of the graph are determined. Finally some results on the matching number of graphs are obtained.

ARTICLE HISTORY

Received 20 December 2019
Revised 29 June 2020
Accepted 11 July 2020

KEYWORDS

Matching number;
characteristic polynomial;
nullity

2000 MATHEMATICS SUBJECT CLASSIFICATIONS

05C70; 05C50; 05C17; 05C31;
05C05

1. Significance of the work

Graph theory is getting more and more popular each day due to its numerous applications and techniques in many areas including some industrial applications. Therefore, graph theory is getting more importance and dealt with by new methods. In this paper, we consider the notion of matching number in relation with the nullity of the graph and study some spectral properties of the graph to obtain some formulae and inequalities for matching number by means of the maximum index of all nonzero coefficients and maximum index of all even nonzero coefficients in the characteristic polynomial by algebraic methods. This method is used for the first time to obtain the matching number of a graph.

2. Introduction

Graph matching has extensive usage area in mathematical modelling as it makes easier to solve some difficult problems saving time and money. Several scientific fields specially computer science use the idea of graph matching in lots of research such as object recognition, object tracking, 3D-modelling, etc., [1]. For 3D-models, similarity measurements of two models can be transformed to a graph matching problem that can easily be solved mathematically, [2]. Moreover, various difficult problems have already been considered as graph theory problems by converting the problem into a graph model to illustrate it mathematically. Some such problems are the Chinese postman problem, travelling salesman problem, utilities problem and Königsberg bridge

problem, and especially in the last years, many problems from molecular chemistry. Besides, there are many more real life problems that can be solved by taking the advantages of graph matchings as above.

In this paper, all graphs that we consider are undirected and simple. G is simple means that G has neither any loops nor multiple edges. Let $G = (V, E)$ be a graph with vertex set V and edge set E . A graph without any edges is called a null graph. Any two points joined by an edge are called adjacent, and each of them is incident with the edge joining them. A subgraph of G is a graph which consists of a subset of V as its vertex set and a subset of E as its edge set. A component of G is a maximal connected subgraph. G is connected if and only if it has exactly one component. That is, a graph is connected iff there is a path between any pair of its vertices. Note that a subgraph G' of G is called an elementary subgraph if every component of G' is either an edge or a cycle. A spanning subgraph G' of a graph G is a subgraph with the same set of vertices as G . A graph with no cycle is called acyclic. A matching M in a graph G is a set of edges such that no two have a vertex in common. A matching that covers every vertex of G is called perfect matching and it is well known that a graph that contains a perfect matching has an even number of vertices. A maximum matching is a matching with the maximum possible number of edges. The number of edges that exist in the maximum matching of G is called matching number of G and in this paper we denote it with $\nu(G)$. A matching having i edges will be called an i -matching. Let $p(G, i)$ be the number of

i -matchings of G , for $1 \leq i \leq \lfloor n/2 \rfloor$ where n denotes the order of the graph G . Moreover, $p(G, 0)$ is designated to be 1. Matching polynomials were defined by Cvetković, Doob, Torgašev in 1988 as

$$M_G(x) = \sum_{i=0}^{\lfloor n/2 \rfloor} (-1)^i p(G, i) x^{n-2i}, \quad (1)$$

see [3, 4]. In [5, 6], the complexity of graphs was studied and in [7], total edge irregularity strength of graphs was discussed. Several other properties of graphs have been investigated in [8–10].

We shall denote the adjacency matrix of G by $A(G)$ where $A(G) = [a_{ij}]_{n \times n}$ is determined by the adjacency of vertices as follows:

$$a_{ij} = \begin{cases} 1, & \text{if the vertices } v_i \text{ and } v_j \text{ are adjacent} \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

$A(G)$ is a symmetric $n \times n$ matrix.

Recall that the characteristic polynomial of a matrix B is given by $|xI - B|$ and similarly, the characteristic polynomial $P_G(x)$ of a graph G is defined by $P_G(x) = |xI - A(G)|$. We can also define the characteristic polynomial of G by means of adjacency matrix $A(G)$ as follows: $P_G(x) = x^n + c_1 x^{n-1} + c_2 x^{n-2} + \dots + c_{n-1} x + c_n$. In 1962, Harary gave the relation

$$|A(G)| = \sum (-1)^{n-c_-(G')-c_0(G')} 2^{c_0(G')} \quad (3)$$

where the summation is taken over all spanning elementary subgraphs G' of G , and $c_-(G')$ and $c_0(G')$ are defined as the number of components in a subgraph G' which are edges and cycles, respectively.

The coefficients of the characteristic polynomial are given by

$$c_k = \sum (-1)^{c_-(G') + c_0(G')} 2^{c_0(G')} \quad (4)$$

where the summation is taken over all elementary subgraphs G' with k vertices for $1 \leq k \leq n$.

There are some methods for determining the matching number of a graph in literature. Usually nullity, spanning trees and several graph parts are used to do this, see e.g. [11–13]. In [14], the authors studied the relations between nullity and matching numbers of unicyclic graphs. In this paper we study the relation between characteristic polynomial and matching number of any graph G and extend a result given for unicyclic graphs in [14] to all graphs. Here, as a new method, the conditions for calculating the matching number of a graph by means of the coefficients of the characteristic polynomial of the graph are determined. Also in some cases, we give formulae to calculate matching number by means of $P_G(x)$. Finally some results on the matching number of graphs are obtained.

3. Matching number by means of characteristic polynomial of graphs

Let n be the number of vertices of G . From now on we shall denote by q^{end} and q^{even} the maximum index of all nonzero coefficients and the maximum index of all even nonzero coefficients in $P_G(x)$, respectively. Here $q^{end} \leq n$, $q^{even} \leq n$. We begin with two theorems to comprehend some of the notions. Recall that all of our graphs are simple.

Theorem 3.1: Let G be a graph with n vertices and edge set $E(G)$. Then

$$\nu(G) > \frac{q^{end}}{2} \quad \text{implies that } \det(A(G)) = 0. \quad (5)$$

Proof: Let $\nu(G) = j$. There is at least an elementary subgraph that consists of j disjoint edges. Since $\nu(G) = j > q^{end}/2$ and $n \geq 2j$, it is clear that $n > q^{end}$. Also, we know that q^{end} is the maximum index of all nonzero coefficient of $P_G(x)$. Hence $c_n = 0$ and by the definition of c_n , we have $\det(A(G)) = 0$. ■

Theorem 3.2: Let G be a graph with n vertices where n is even and edge set $E(G)$. Then $\det(A(G)) = 0$ and G has a perfect matching implies that $\nu(G) > q^{end}/2$.

Proof: Let G have a perfect matching. Then there is a subgraph G' that consists of only disjoint edges and since $|V(G)| = n$, G' has $n/2$ edges and it is equal to $\nu(G)$. We know that $c_n = (-1)^n \det(A(G))$ and by the hypothesis $\det(A(G)) = 0$, so the maximum index of all nonzero coefficients that exist in $P_G(x)$ will be $q^{end} < n$, namely, $\nu(G) > q^{end}/2$. ■

Example 3.1: Let us consider a graph G which is a simple graph with 10 vertices as in Figure 1.

The characteristic polynomial of this graph is

$$\begin{aligned} P_G(x) &= x^{10} - 12x^8 - 4x^7 + 46x^6 + 30x^5 \\ &\quad - 55x^4 - 54x^3 - 4x^2 + 4x \\ &= x^{10} + c_1 x^9 + c_2 x^8 + c_3 x^7 + c_4 x^6 + c_5 x^5 \\ &\quad + c_6 x^4 + c_7 x^3 + c_8 x^2 + c_9 x + c_{10}. \end{aligned} \quad (6)$$

We see that $c_{10} = \det(A(G)) = 0$ and G has a perfect matching. Therefore by Theorem 3.2, we obtain $\nu(G) = 5$ and $q^{end} = 9$ which satisfies Theorem 3.2.

Lemma 3.3: Let G be an l -cycle graph, that is $G = C_l$. Then

$$\det(A(G)) = \begin{cases} 2, & l \equiv 1 \pmod{4} \text{ or } l \equiv 3 \pmod{4} \\ -4, & l \equiv 2 \pmod{4} \\ 0, & l \equiv 0 \pmod{4}. \end{cases} \quad (7)$$

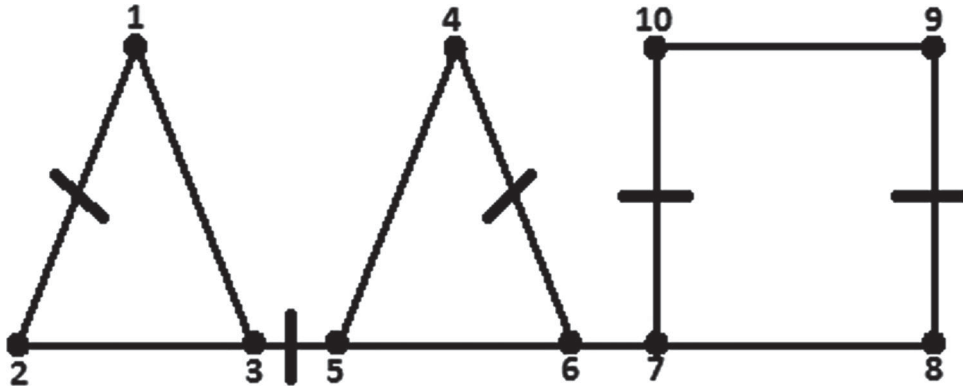


Figure 1. A graph G with 10 vertices.

Proof: Let G be an l -cycle graph. The adjacency matrix of G is

$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

We calculate the determinant of $A(G)$ by means of the elementary row operations $R_1 \longleftrightarrow R_2$, $R_2 \longleftrightarrow R_3, \dots, R_{n-1} \longleftrightarrow R_n$.

Now for $l \equiv 2 \pmod{4}$, we first apply $-R_1 + R_{n-1} \rightarrow R_{n-1}$, $-R_2 + R_n \rightarrow R_n$. After these operations, we respectively apply $R_3 + R_{n-1} \rightarrow R_{n-1}$, $R_4 + R_n \rightarrow R_n$, up to $R_{n-3} + R_{n-1} \rightarrow R_{n-1}$, $R_{n-2} + R_n \rightarrow R_n$.

Consequently, we obtain an upper triangular matrix and the determinant of this matrix is

$$\det(A(G)) = (-1)^{4l+1} 2^2 = -4. \quad (8)$$

For $l \equiv 0 \pmod{4}$, the proof is similar.

Thirdly, for $l \equiv 1 \pmod{4}$, we apply $-R_1 + R_{n-1} \rightarrow R_{n-1}$, $-R_2 + R_n \rightarrow R_n$, and after these we apply $R_3 + R_{n-1} \rightarrow R_{n-1}$, $R_4 + R_n \rightarrow R_n$, up to the operations $-R_{n-4} + R_{n-1} \rightarrow R_{n-1}$, $-R_{n-3} + R_n \rightarrow R_n$. Finally we apply $R_{n-2} + R_{n-1} \rightarrow R_{n-1}$ and then $R_{n-1} + R_n \rightarrow R_n$ to get

$$\det(A(G)) = (-1)^{4l} 2 = 2. \quad (9)$$

Finally, for $l \equiv 3 \pmod{4}$, the proof is similar. ■

In [15], another proof of Lemma 3.3 was given.

In the next two theorems, we give a way of calculation $\nu(G)$ and show the existence of the maximum q^{even} for the graphs that have some conditions, respectively.

Theorem 3.4: Let G be a simple graph with n vertices that is allowed to contain only $4l$ -cycles and no other cycle and let $\nu(G) = j$. If at least one elementary subgraph G' with only j disjoint edges and any even number of edges do not

form a spanning elementary subgraph of any $4l$ -cycle in G can be found, then

$$\nu(G) = \frac{q^{even}}{2} = \frac{q^{end}}{2}. \quad (10)$$

Proof: By the Lemma 3.3, since the determinants of adjacency matrices of $4l$ -cycles are zero, the rows of adjacency matrices of $4l$ -cycles which correspond to the vertices are linearly dependent. Thereby, if any even number of edges we count in $\nu(G)$ form a spanning elementary subgraph of any $4l$ -cycle that exist in G in all cases, then the sum of all $2j \times 2j$ principal minors are zero because all elementary subgraphs with $2j$ vertices have even number of edges that form a spanning elementary subgraph of any $4l$ -cycles in G . By the definition of c_k , we know that $c_{2j} = c_{q^{even}} = 0$. However, if there is at least an elementary subgraph with j disjoint edges so that any even number of edges do not form a spanning elementary subgraph of any $4l$ -cycle in G , then $c_{2j} = c_{q^{even}} \neq 0$. Hence $\nu(G) = j = 2j/2 = q^{even}/2$. Finally, since G is allowed to contain only $4l$ -cycles as cycle, we observe that $\nu(G) \neq q^{even}/2$. Consequently, we see that

$$\nu(G) = \frac{q^{even}}{2}. \quad (11)$$

Moreover, by the definition of c_k we know that c_k is constituted by elementary subgraphs that consist of disjoint edges and cycles. Since G is allowed to contain only $4l$ -cycles then for every nonzero c_k , k must be even. Therefore $q^{even} = q^{end}$. As a result, we get $q^{even}/2 = q^{end}/2$. ■

Corollary 3.5: Let G be a simple graph with n vertices that is allowed to contain only $4l$ -cycles as cycle. Let the characteristic polynomial of G be

$$\begin{aligned} P_G(x) &= |xI - A(G)| \\ &= x^n + c_1 x^{n-1} + c_2 x^{n-2} + \dots + c_{n-1} x + c_n. \end{aligned} \quad (12)$$

If at least an elementary subgraph G' with q^{even} vertices that consists of only disjoint edges and any even number

of edges do not form a spanning elementary subgraph of a $4l$ -cycle in G can be found, then

$$c_{q^{even}} \neq 0. \quad (13)$$

Proof: By Theorem 3.4 and Lemma 3.3, we tend to the sum of $q^{even} \times q^{even}$ principal minors of $A(G)$, and hence the result is seen. ■

Example 3.2: Let us consider a simple graph G with 8 vertices as in Figure 2.

Then the characteristic polynomial of G will be

$$\begin{aligned} P_G(x) &= x^8 - 8x^6 + 16x^4 - 8x^2 + 1 \\ &= x^8 + c_1x^7 + c_2x^6 + c_3x^5 + c_4x^4 + c_5x^3 \\ &\quad + c_6x^2 + c_7x + c_8. \end{aligned} \quad (14)$$

Since we have found at least one subgraph G' with 4 discrete edges which are shown by drawing a small line piece on them in Figure 2, and in G' any even number of edges can not form a spanning subgraph of any $4l$ -cycle in G , we get $c_8 = 1 \neq 0$. Besides $\nu(G) = q^{even}/2 = \frac{8}{2} = 4$ because we have found at least one elementary subgraph with only $\nu(G)$ disjoint edges.

Observe that if we rewrite the result of Theorem 3.4 as $\nu(G) = q^{even}/2$, then under this condition, in the hypotheses of Theorem 3.4 and Corollary 3.5, one can also make the alteration that G is a unicyclic graph instead of the condition that G is allowed to contain only $4l$ -cycles as cycle. Moreover, if G is a unicyclic graph and the cycle is not a $4k$ -cycle, then $\nu(G) = q^{even}/2$ is satisfied without any further conditions. To show this, we have to work out the $(4l+1)$ -cycles, $(4l+2)$ -cycles and $(4l+3)$ -cycles as we proved the result for $4l$ -cycles already above.

We know that any elementary or spanning elementary subgraph of a unicyclic graph that contributes to

the quantity $c_{q^{even}}$ cannot have any odd cycle component, so the result is obvious for $(4l+1)$ -cycles and $(4l+3)$ -cycles. Thus, we must only mention $(4l+2)$ -cycles. It is clear that when a $(4l+2)$ -cycle component exists in an elementary or spanning elementary subgraph as cycle, it contributes a negative amount to $c_{q^{even}}$ since the contribution of this cycle to the number of components of corresponding subgraph is 1. Besides, if we take only the edges in the $(4l+2)$ -cycle component in an elementary or spanning elementary subgraph as edges, then it contributes a negative number to $c_{q^{even}}$ since the contribution of the edges to the number of components of the corresponding subgraph would be $2l+1$. So any number $c_{q^{even}}$ that must be nonzero in $P_G(x)$, thanks to elementary or spanning elementary subgraphs, cannot be zero because of the same sign. Hence, we get the required result.

It is important to point out that in the case of G is unicyclic, we have to change the result of Theorem 3.4 because if G is a unicyclic graph and the cycle of graph is a $(4l+1)$ -cycle or $(4l+3)$ -cycle, then q^{end} would be odd, so in this case, we get $q^{end} \neq q^{even}$.

Let us define some notions that are used in the forthcoming theorems.

Let G be a simple graph with $|V(G)| = n$, $|E(G)| = m$, q_i^{even} be the maximum index of all even nonzero coefficients that exist in the characteristic polynomial of the i th component of G , q_i^{end} be the maximum index of all nonzero coefficients that exist in the characteristic polynomial of the i th component of G , $\zeta(G)$ be the number of components that exist in G , $\nu(C_i)$ be the matching number of C_i and $\theta(G)$ be the number of components of G that verify the condition that $(q_i^{end} - 1)/2 = \nu(C_i)$.

If a graph G contains no two cycles with a common vertex, then we call it vertex-disjoint cycle graph. In the next theorems, we use a cycle removing process. When we remove a cycle, we delete all edges that are incident to the deleted vertices of the cycle. This removing

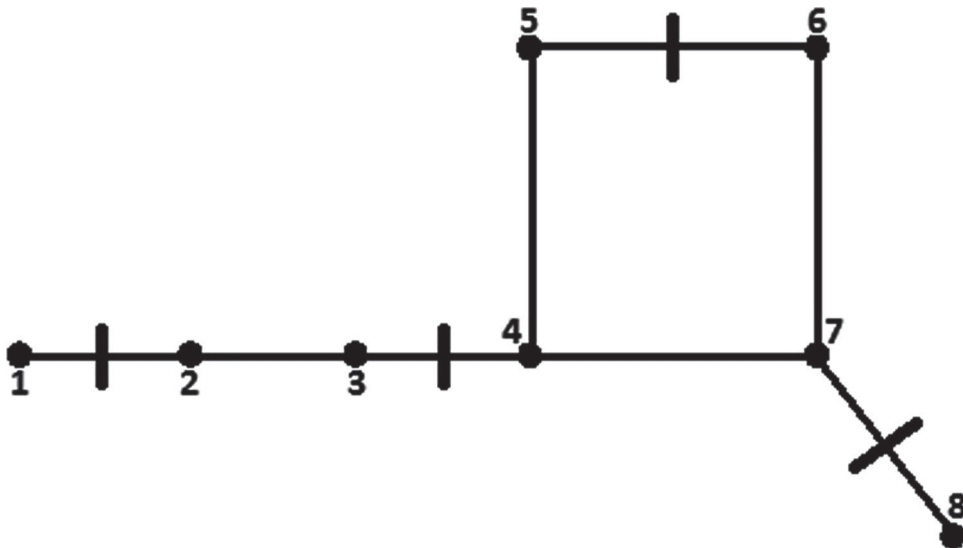


Figure 2. A graph G with 8 vertices.

that $(q_i^{end} - 1)/2 = v(C_i)$ implies that

$$v(G) + \left\lfloor \frac{\varsigma(G)}{2} \right\rfloor = \left\lfloor \frac{q^{end}}{2} \right\rfloor. \quad (17)$$

Proof: Let G be a graph. If G is connected, then $v(C_i) = v(G)$, $q_i^{end} = q^{end}$ and $\lfloor \varsigma(G)/2 \rfloor = 0$. Hence as $(q_i^{end} - 1)/2 = v(C_i)$, we have $(q^{end} - 1)/2 = v(G)$ which implies the required result.

If G is disconnected, then G has at least two components. Let for every C_i , $(q_i^{end} - 1)/2$ be equal to $v(C_i)$. Since q_i^{end} is odd, if $\varsigma(G)$ is even, then $\sum_{i=1}^{\varsigma(G)} q_i^{end}$ is also even so that $q^{end} = \sum_{i=1}^{\varsigma(G)} q_i^{end}$ is even. However for every component, a vertex participating to q_i^{end} that does not exist in $v(C_i)$ and therefore in $v(G)$. Hence we have additional vertices whose number is equal to the component number of G . Moreover, here $\varsigma(G)$ and q^{end} are even, so we can utilize the greatest integer function. As a result, we get the required result.

If $\varsigma(G)$ is odd, $\sum_{i=1}^{\varsigma(G)} q_i^{end}$ is odd so that $q^{end} = \sum_{i=1}^{\varsigma(G)} q_i^{end}$ is odd. As above, for every component, a vertex adds 1 to q^{end} . Here only an even number of such vertices exist in $v(G)$, so we must use the greatest integer function as above. Consequently, we have the required result. ■

Theorem 3.10: Let G be a simple graph with n vertices. For every component C_j such that $v(C_j) = q_j^{even}/2$, except the components that satisfy the condition that $(q_i^{end} - 1)/2 = v(C_i)$, then

$$v(G) + \left\lfloor \frac{\theta(G)}{2} \right\rfloor = \left\lfloor \frac{q^{end}}{2} \right\rfloor. \quad (18)$$

Proof: Let G be a simple graph with n vertices. In Theorem 3.9, every component satisfies the condition that $(q_i^{end} - 1)/2 = v(C_i)$, but here the number of components of G that satisfy the given condition is equal to $\theta(G)$. The proof is similar to the proof of Theorem 3.9, except for some differences: In the statement of Theorem 3.9, it is said that every component C_i satisfies the condition. Hence, by means of this condition, $c_{q^{end}}$ must be nonzero in $P_G(x)$ because of the fact that for the elementary or spanning elementary subgraphs, it cannot be zero. So, in our case, there can be some components that do not satisfy the condition. Because of this, we need an additional condition. Since the components different than the ones that satisfy the condition $(q_i^{end} - 1)/2 = v(C_i)$ satisfy $v(C_j) = q_j^{even}/2$, $c_{q^{end}}$ must be nonzero in $P_G(x)$ as the elementary or spanning elementary subgraphs cannot be zero. Hence, by Theorem 3.9, the proof is concluded. ■

Corollary 3.11: Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. Except the $\theta(G)$ components, for every component C_j , $v(C_j) = q_j^{even}/2$ and $\theta(G) \geq 2$ implies that $v(G) < \lfloor q^{end}/2 \rfloor$.

Theorem 3.12: Let G be a simple graph with $n \leq 8$ vertices and m edges. Let x, y, z be the number of 4-, 6- and 8-cycles in G , respectively. Let t and u be the number of remaining edges after deleting the vertices of corresponding 4- and 6-cycles in G , respectively. Let a be the number of 2-matchings in the remaining graph after deleting the vertices of corresponding 4-cycle in G . Let b, d and f be the number of combinations of all disjoint 3-cycles, 3- and 5-cycles, and two 4-cycles in G , respectively. Then

$$-p(G, 1) = c_2 = m, \quad (19)$$

$$p(G, 2) = c_4 + 2 \sum_{i=1}^x 1, \quad (20)$$

$$-p(G, 3) = c_6 + 2 \sum_{i=1}^y 1 - 2 \sum_{i=1}^x t - 4 \sum_{i=1}^b 1, \quad (21)$$

$$p(G, 4) = c_8 + 2 \sum_{i=1}^z 1 + 2 \sum_{i=1}^x a - 2 \sum_{i=1}^y u - 4 \sum_{i=1}^d 1 - 4 \sum_{i=1}^f 1. \quad (22)$$

Proof: All possible edge numbers are 2, 4, 6 and 8. We can express these numbers as a summation of our components that are edges and cycles. The expressions are as follows:

$$2 = 0 + 2, \quad (23)$$

$$4 = 2 + 2 = 0 + 4, \quad (24)$$

$$6 = 2 + 2 + 2 = 2 + 4 = 3 + 3 = 0 + 6 \quad (25)$$

$$8 = 2 + 2 + 2 + 2 = 2 + 2 + 4 = 2 + 6 = 4 + 4 = 3 + 5 = 0 + 8. \quad (26)$$

Hence the proof follows. ■

4. Results and discussion

Spectrum of a graph is the set of eigenvalues of the characteristic polynomial of the graph obtained by means of the adjacency matrix. The branch of graph theory dealing with the spectral study of graphs is therefore named as the spectral graph theory. The graph energy was defined by Gutman and used to obtain certain physico-chemical properties of molecular graphs. Matching is another important notion related to graphs with numerous applications. In this paper, we gave relations between these two important topics in connection with graphs. Several general results giving these relations are obtained. Differently than the existing results in literature, we obtained results on the matching number of a graph by means of the characteristic polynomial. A result given for only unicyclic graphs is generalized. There are some methods for determining the matching number of a graph in literature. Usually nullity, spanning trees and several graph parts are used

to do this. Here, as a new method, the conditions for calculating the matching number of a graph by means of the coefficients of the characteristic polynomial of the graph are determined. We use spanning elementary subgraphs of unicyclic graphs to do the calculations. The same technique can be extended to graph operations and derived graphs such as the line, total, subdivision, middle, centre and Mycielskian graphs. A lower bound for the matching number of a graph is given. It is shown that if the matching number is greater than the maximum index of the nonzero coefficients, then the determinant of the adjacency matrix is zero. Similarly, it is proven that if this determinant equals to 0 and the graph has a perfect matching, the matching number has a lower bound. The value of this determinant is given in terms of cycle lengths.

5. Conclusions

A new method to calculate the matching number of a graph by means of the characteristic polynomial and its coefficients is developed by deleting graph parts from the graph, especially by deleting the vertices belonging to the cycles of several lengths. It is possible to apply this method to many other graph classes, graph operations and derived graphs. As the nullity depends on other graph parameters, it may also be possible to relate the matching number with those parameters using this deletion method.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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